B.SC. FOURTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics | **Course ID: 42114**

Course Code: SH/MTH /404/GE-4 | **Course Title: Differential Equations and Vector Calculus**

Full Marks: 40 | **Time: 2 Hours**

The figures in the margin indicate full marks

Unless otherwise mentioned the symbols have their usual meaning

1. **Answer *any FIVE* of the following questions:** **2×5=10**

a) Illustrate by an example that a continuous function may not satisfy Lipschitz condition on a rectangle.

b) If $y_1 = sin3x$ and $y_2 = cos3x$ then find the Wronskian of y_1 and y_2.

c) Solve: $4\frac{d^2y}{dx^2} + \frac{dy}{dx} - 3y = 0$.

d) Test the continuity of the vector function $\vec{f}(t) = |t|\hat{\imath} - \sin(t)\hat{\jmath} + (1 + cost)\hat{k}$ at t=0.

e) Show that x=0 is a regular singular point of the differential equation $2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x + 1)y = 0$.

f) Find the equilibrium points of the system $\frac{dx}{dt} = y^2 - 7x + 12, \frac{dy}{dt} = x - y$.

g) Use Wronskian to show that the functions $f(x) = x,\ g(x) = x^2,\ h(x) = x^3$ are independent.

h) Find equilibrium points of the system $\dot{x} = x + x^3$.

2. **Answer *any FOUR* of the following questions:** **4×5=20**

a) Solve: $x^2\frac{d^2y}{dx^2} - 2y = x^3$ by the method of variation of parameters.

b) Prove that the two solutions $y_1(x)$ and $y_2(x)$ of the equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0, x \in (a, b)$ are linearly independent if and only if their Wronskian is not zero at some point x_0 on (a, b).

c) Solve the equation $(D^2 - 2D + 1)y = xe^x$ by the method of undetermined coefficients.

d) Are these three vectors $7\hat{\imath}$- $9\hat{\jmath} + 11\hat{k}$, $3\hat{\imath}$+$\hat{\jmath} - 5\hat{k}$, $5\hat{\imath}$-$21\hat{\jmath} + 37\hat{k}$ coplanar or not?

e) Find the solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ satisfying the conditions y(0)=1 and y$'$ (0)=4.

f) Find the Power series solution of the differential equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = 0$ about $x = 0$.

3. Answer *any ONE* of the following questions: **10×1=10**

a) i) Solve: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$ 5

ii) A particle moves along the curve $x = e^{-2t}, y = 2\cos 2t, z = 2\sin 3t$. Determine the velocity and the acceleration of the particle at any time t and their magnitude at $= 0$. 3

iii) Solve: $\frac{dx}{x^2+y^2+yz} = \frac{dy}{x^2+y^2-xz} = \frac{dz}{z(x+y)}$ 2

b) i) Find the solution of the system: $\frac{dx}{dt} = x - 5y, \quad \frac{dy}{dt} = x - 3y$ satisfying the initial conditions $x(0) = 1,\ y(0) = 1$. Describe the beheaviour of the solution as $t \to \infty$. 6

ii) Evaluate $\int F.d\vec{r}$ where $F(x,y) = (6x-2y)\hat{\imath} + x^2\hat{\jmath}$ for the curve C where C is the line segment from (6,−3) to (6,3). 4

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